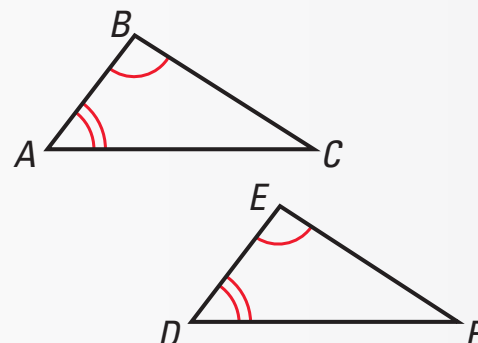


THEOREM

THEOREM 4.3 *Third Angles Theorem*

If two angles of one triangle are congruent to two angles of another triangle, then the third angles are also congruent.

If $\angle A \cong \angle D$ and $\angle B \cong \angle E$,
 then $\angle C \cong \angle F$.

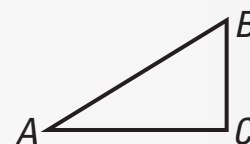


THEOREM

THEOREM 4.4 *Properties of Congruent Triangles*

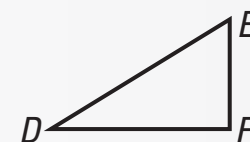
REFLEXIVE PROPERTY OF CONGRUENT TRIANGLES

Every triangle is congruent to itself.



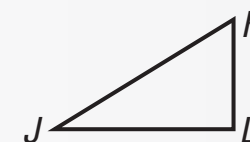
SYMMETRIC PROPERTY OF CONGRUENT TRIANGLES

If $\triangle ABC \cong \triangle DEF$, then $\triangle DEF \cong \triangle ABC$.



TRANSITIVE PROPERTY OF CONGRUENT TRIANGLES

If $\triangle ABC \cong \triangle DEF$ and $\triangle DEF \cong \triangle JKL$, then $\triangle ABC \cong \triangle JKL$.

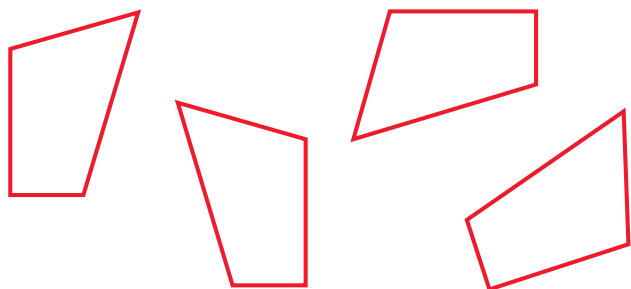


Geometry Date _____ 4.2 Notes

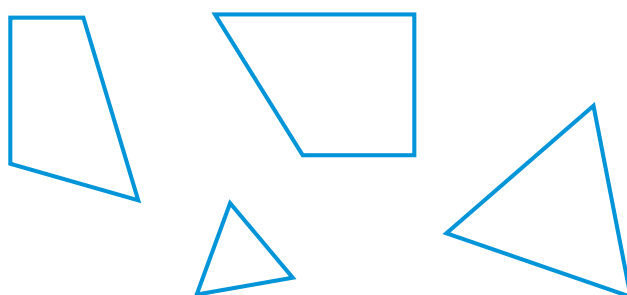
Congruence and Triangles (pp 202–205)

Two geometric figures are *congruent* if they have exactly the same size and shape. Each of the red figures is congruent to the other red figures. None of the blue figures is congruent to another blue figure.

Congruent



Not congruent



When two figures are **congruent**, there is a correspondence between their angles and sides such that **corresponding angles** are congruent and **corresponding sides** are congruent. For the triangles below, you can write $\triangle ABC \cong \triangle PQR$, which is read “triangle ABC is congruent to triangle PQR .” The notation shows the congruence and the correspondence.

Corresponding angles

$$\angle A \cong \angle P$$

$$\angle B \cong \angle Q$$

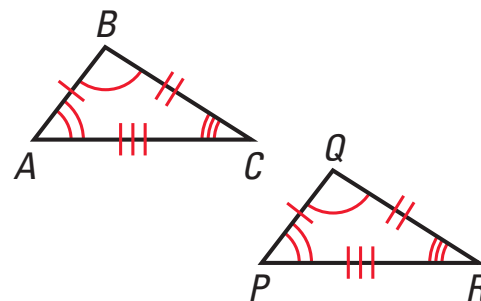
$$\angle C \cong \angle R$$

Corresponding sides

$$\overline{AB} \cong \overline{PQ}$$

$$\overline{BC} \cong \overline{QR}$$

$$\overline{CA} \cong \overline{RP}$$



There is more than one way to write a congruence statement, but it is important to list the corresponding angles in the same order. For example, you can also write $\triangle BCA \cong \triangle QRP$.



Geometry Date _____ 4.2 Notes

Congruence and Triangles (pp 202–205)

EXAMPLE 1 Naming Congruent Parts

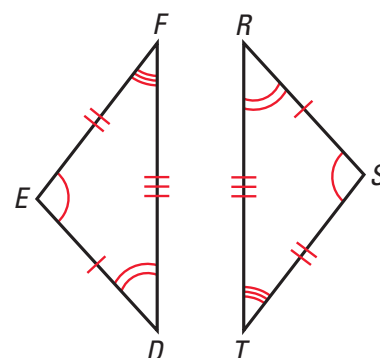
The congruent triangles represent the triangles in the photo above. Write a congruence statement. Identify all pairs of congruent corresponding parts.

SOLUTION

The diagram indicates that $\triangle DEF \cong \triangle RST$.
The congruent angles and sides are as follows.

Angles: $\angle D \cong \angle R$, $\angle E \cong \angle S$, $\angle F \cong \angle T$

Sides: $\overline{DE} \cong \overline{RS}$, $\overline{EF} \cong \overline{ST}$, $\overline{FD} \cong \overline{TR}$

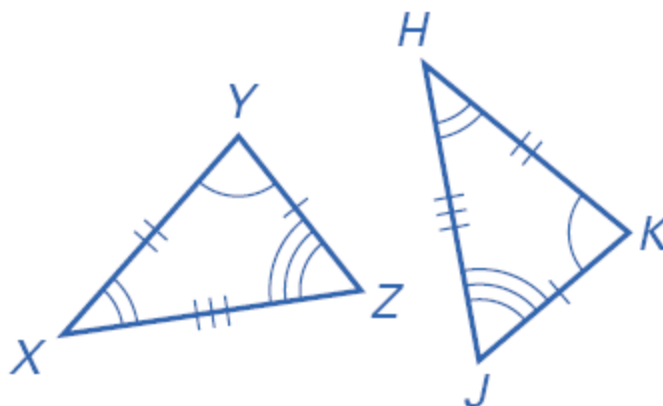


STUDENT HELP

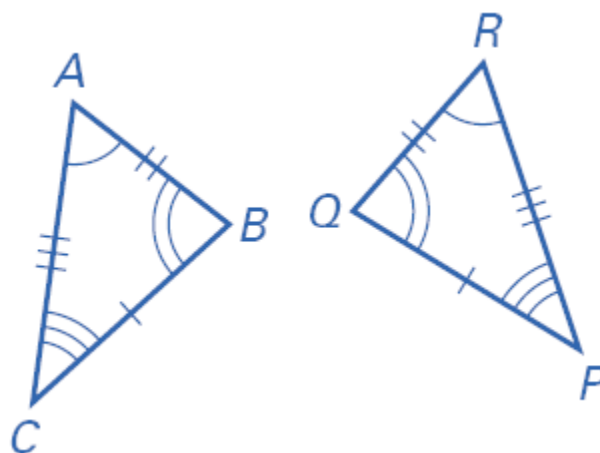
Study Tip

Notice that single, double, and triple arcs are used to show congruent angles.

1. Write a congruence statement for the triangles. Identify all pairs of congruent corresponding parts.

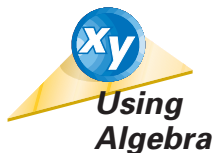


2. Identify all pairs of congruent corresponding parts.



Geometry Date_____ 4.2 Notes

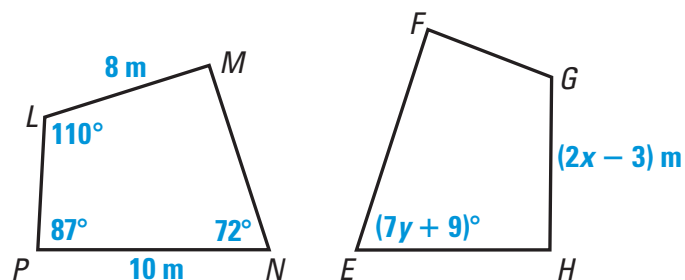
Congruence and Triangles (pp 202–205)



EXAMPLE 2 Using Properties of Congruent Figures

In the diagram, $NPLM \cong EFGH$.

- Find the value of x .
- Find the value of y .



SOLUTION

- You know that $\overline{LM} \cong \overline{GH}$.
So, $LM = GH$.

$$8 = 2x - 3$$

$$11 = 2x$$

$$5.5 = x$$

- You know that $\angle N \cong \angle E$.
So, $m\angle N = m\angle E$.

$$72^\circ = (7y + 9)^\circ$$

$$63 = 7y$$

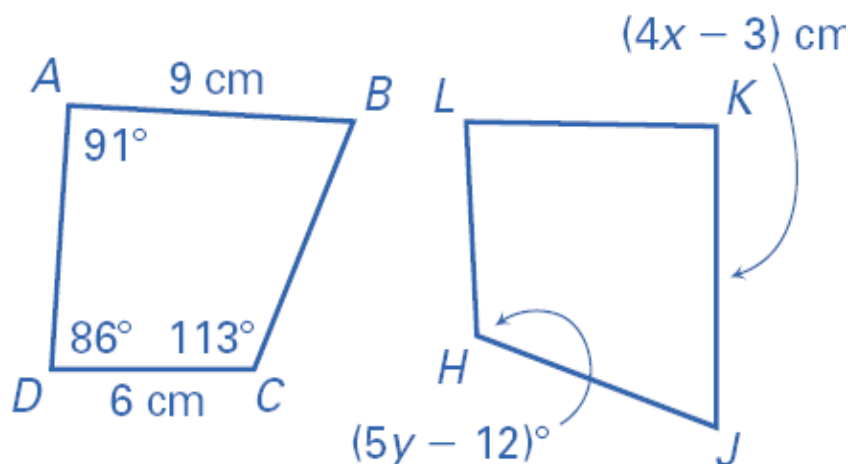
$$9 = y$$

Examples.

In the diagram $ABCD \cong KJHL$.

- Find the value of x .

- Find the value of y .

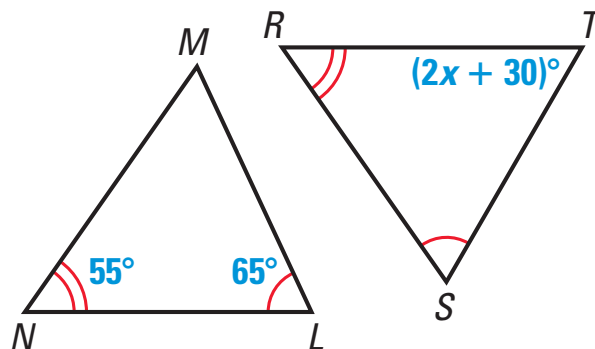


EXAMPLE 3 *Using the Third Angles Theorem*

Find the value of x .

SOLUTION

In the diagram, $\angle N \cong \angle R$ and $\angle L \cong \angle S$.
 From the Third Angles Theorem, you know
 that $\angle M \cong \angle T$. So, $m\angle M = m\angle T$.
 From the Triangle Sum Theorem,
 $m\angle M = 180^\circ - 55^\circ - 65^\circ = 60^\circ$.



$$m\angle M = m\angle T$$

Third Angles Theorem

$$60^\circ = (2x + 30)^\circ$$

Substitute.

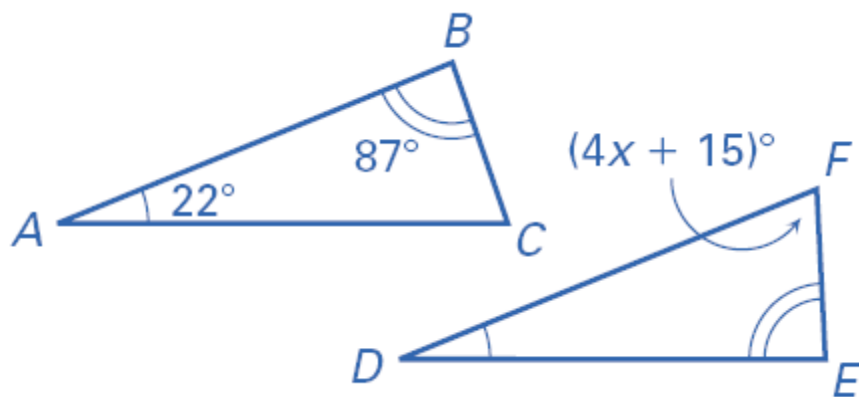
$$30 = 2x$$

Subtract 30 from each side.

$$15 = x$$

Divide each side by 2.

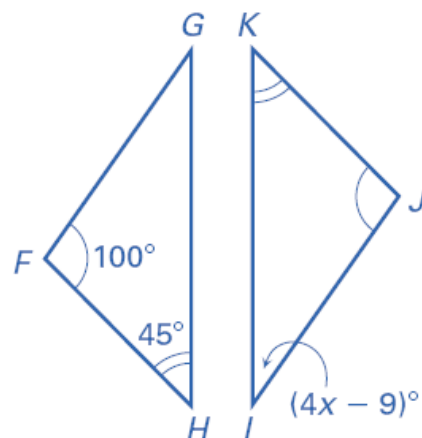
5. Find the value of x .



Geometry Date_____ 4.2 Notes

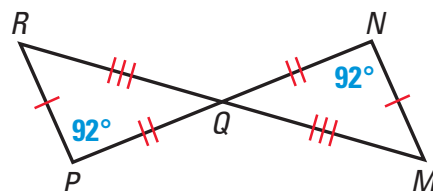
Congruence and Triangles (pp 202–205)

6. Find the value of x .



EXAMPLE 4 Determining Whether Triangles are Congruent

Decide whether the triangles are congruent.
Justify your reasoning.



SOLUTION

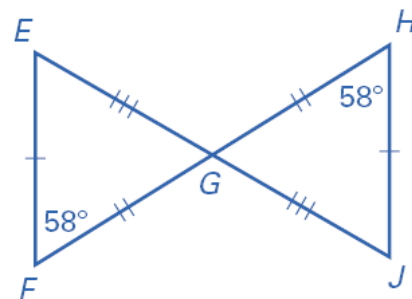
Paragraph Proof From the diagram, you are given that all three pairs of corresponding sides are congruent.

$$\overline{RP} \cong \overline{MN}, \overline{PQ} \cong \overline{NQ}, \text{ and } \overline{QR} \cong \overline{QM}$$

Because $\angle P$ and $\angle N$ have the same measure, $\angle P \cong \angle N$. By the Vertical Angles Theorem, you know that $\angle PQR \cong \angle NQM$. By the Third Angles Theorem, $\angle R \cong \angle M$.

► So, all three pairs of corresponding sides and all three pairs of corresponding angles are congruent. By the definition of congruent triangles, $\triangle PQR \cong \triangle NQM$.

7. Decide whether the triangles are congruent. Justify your reasoning.



Geometry Date _____ 4.2 Notes

Congruence and Triangles (pp 202–205)

EXAMPLE 5 Proving Two Triangles are Congruent

FOCUS ON APPLICATIONS



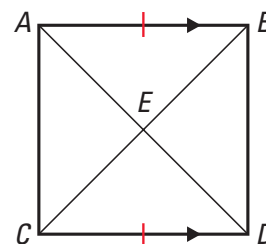
REAL LIFE TRIANGULAR STAMP

When these stamps were issued in 1997, Postmaster General Marvin Runyon said, "Since 1847, when the first U.S. postage stamps were issued, stamps have been rectangular in shape. We want the American public to know stamps aren't 'square.'"

The diagram represents the triangular stamps shown in the photo. Prove that $\triangle AEB \cong \triangle DEC$.

GIVEN ▶ $\overline{AB} \parallel \overline{DC}$, $\overline{AB} \cong \overline{DC}$,
 E is the midpoint of \overline{BC} and \overline{AD} .

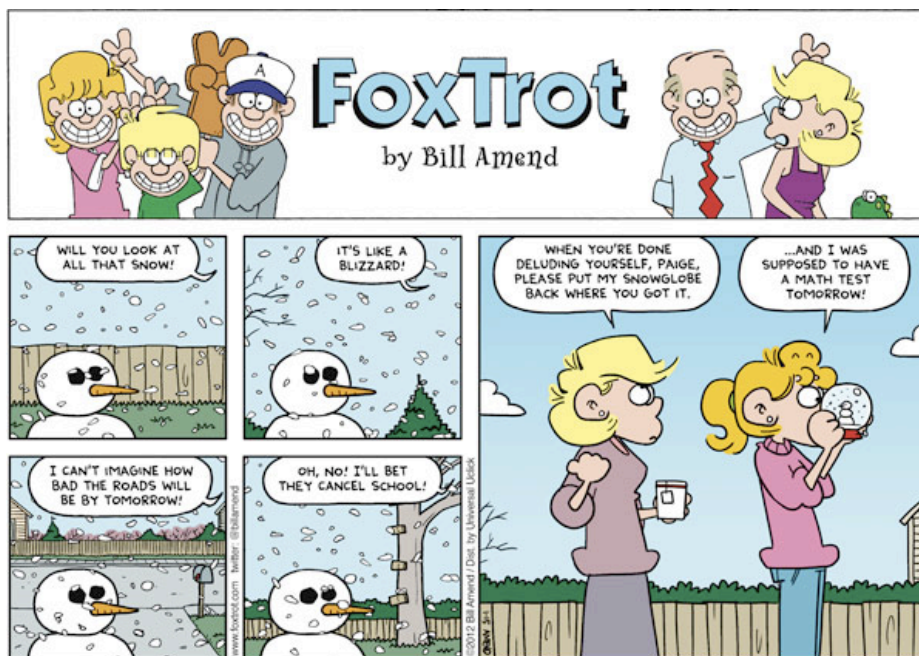
PROVE ▶ $\triangle AEB \cong \triangle DEC$



Plan for Proof Use the fact that $\angle AEB$ and $\angle DEC$ are vertical angles to show that those angles are congruent. Use the fact that \overline{BC} intersects parallel segments \overline{AB} and \overline{DC} to identify other pairs of angles that are congruent.

SOLUTION

Statements	Reasons
1. $\overline{AB} \parallel \overline{DC}$, $\overline{AB} \cong \overline{DC}$	1. Given
2. $\angle EAB \cong \angle EDC$, $\angle ABE \cong \angle DCE$	2. Alternate Interior Angles Theorem
3. $\angle AEB \cong \angle DEC$	3. Vertical Angles Theorem
4. E is the midpoint of \overline{AD} , E is the midpoint of \overline{BC} .	4. Given
5. $\overline{AE} \cong \overline{DE}$, $\overline{BE} \cong \overline{CE}$	5. Definition of midpoint
6. $\triangle AEB \cong \triangle DEC$	6. Definition of congruent triangles



Geometry Date _____ 4.2 Notes Congruence and Triangles (pp 202–205)

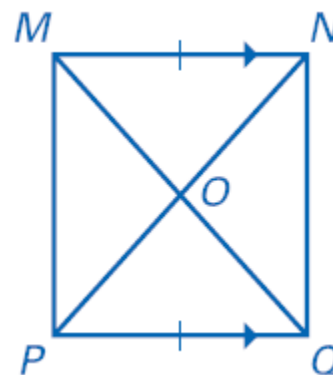
8. Complete the proof.

$$\overline{MN} \cong \overline{QP}$$

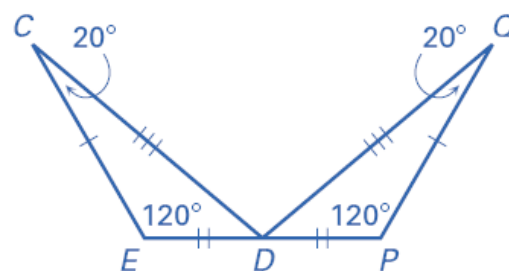
Given: $\overline{MN} \parallel \overline{PQ}$

O is the midpoint of \overline{MQ} & \overline{PN} .

Prove: $\triangle MNO \cong \triangle QPO$



9. Decide whether the triangles are congruent. Justify your reasoning.

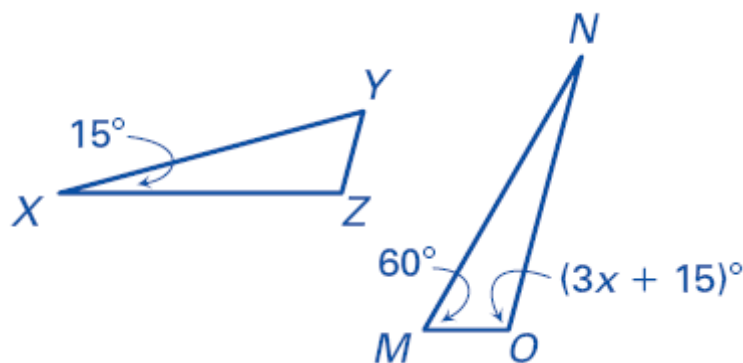


10. _____ Given

$\angle X \cong \angle N$ & $\angle Z \cong \angle O$, find the

value of x .

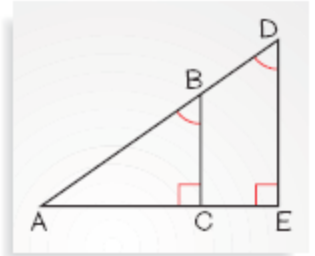
- A. 20°
- B. 30°
- C. 60°
- D. 90°
- E. 105°



Geometry Date_____ 4.2 Notes
Congruence and Triangles (pp 202–205)

On an exam, a student says that $\triangle ABC \cong \triangle ADE$ because the corresponding angles are congruent.

11. How does the student know that the corresponding angles are congruent?



12. Is $\triangle ABC \cong \triangle ADE$? Explain your answer?